Subtle Sources of Error in Laser Trackers Due to Dispersion in the Internal Optical Elements

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ABSTRACT

It is well known that the speed of light depends on the index of refraction of the medium in which the light is propagating. It is also well known that in a dispersive medium, the speed of an amplitude modulated wavefront depends on the group refractive index, i.e., slightly slower than the carrier light. Corrections for the group refractive index in air are typically made for temperature, humidity, and pressure—without which measurements could be in error by tens of parts per million. The internal instrument optical elements are also subject to dispersive effects, which have heretofore been ignored in the literature—and presumably in the design. Note that this is probably because no commercially available optical design software package models amplitude modulated wavefronts. A thought experiment will illustrate the problem. From Fermat's principal, a plane wave intersecting a converging lens bends the wave to converge at a focal point. The lens is shaped such that the propagation time to the focal point is the same for all rays. For example, a ray passing through the outer radius of the lens passes through a thinner section of glass, but must propagate a longer distance to the focus. A ray passing through the center of the lens passes through a thicker section of glass, but propagates through a shorter distance to the focus. However, for optical amplitude modulated (OAM) light, the modulated wavefront, which has two sidebands that propagate at slightly different speeds in a dispersive medium, does not reach the focus at the same time! In other words, there is a slight phase shift in the modulated wavefront between the beam passing through the center of the lens, and the beam passing through the outer radius of the lens. This makes the net phase of the modulated waverfont, as received by a detector at the focal point, dependent on the beam geometry—which most likely depends on distance, due to divergence of the beam. At close range, the majority of the received beam passes through the center of the lens, due to the small beam size. At long range, the received beam passes through the entire lens, due to the expanded beam filling the lens. This source of error can be misinterpreted as being due to distance or power level, when in fact it is the optical design. Spherical, or cat's eye, retroreflectors are also subject to the same source of error. A simple test to measure the errors is proposed.

INTRODUCTION

This is a companion paper to *Aberrations of temporally modulated optical wavefronts in dispersive optical systems* [1] (THE SPIE PAPER), which was drawn to a more theoretical analysis of the physics of optical amplitude modulation (OAM) and ramifications of dispersion, as well as a review of both the patent and non-patent literature. This paper is drawn to the narrower aspect of practical implications for Electronic Distance Measurement (EDM) instrument design and use, i.e., subtle sources of error in EDM.

FUNDAMENTALS OF OPTICAL AMPLITUDE MODULATION

Optics texts deal with spatial modulation, e.g., resolution is typically measured by the Modulation Transfer Function (MTF). No known optics text deals with temporal modulation, so to avoid confusion, the term Optical Amplitude Modulation (OAM) is used to describe temporal modulation of light.

Most EDM instruments are based on laser interferometry, phase measurement of an OAM laser, or a pulsed laser. Other than a simple Michelson laser interferometer, which uses a continuous wave (CW) beam at a single wavelength, all of these architectures use some form of OAM.

It is useful to think of OAM light as an analog to amplitude modulated (AM) radio [2,3]. In AM radio, a carrier radio frequency (RF) signal (typically 540-1,700 kHz) is amplitude modulated by a low frequency audio signal. From simple Fourier analysis, this produces two sidebands at the carrier plus the modulation frequency, and the carrier minus the modulation frequency [4]. Conversely, if two slightly different frequency signals are combined, a modulation frequency beat note signal is produced with a frequency equal to half the difference in frequencies. For example, a 5 kHz modulation signal could be produced by amplitude modulating a 1,700 kHz carrier at 5 kHz, or by somehow producing two signals at 1,695 kHz and 1,705 kHz.

The same principles apply for OAM laser light—except the carrier frequency is typically much, much greater than the modulation frequency. For example, a HeNe laser operates at 632.8 nm wavelength, which corresponds to 474 THz. Typical EDM instruments use modulation frequencies between several MHz and several GHz, i.e., the sidebands are spaced around one part in 500,000 from the carrier frequency—or from a typical optical designer's perspective, there is no difference between the carrier and the sidebands. Conventional optical design software [1] would not account for the sidebands because there would be no chromatic aberration [5], and the sidebands would follow the same ray tracings as the carrier.

Photodetectors are not capable of measuring THz frequency signals directly. It is necessary to heterodyne the information to a much lower RF, or microwave frequency, in order to extract the information. It was shown in THE SPIE PAPER that the relative phase between the two sidebands corresponds to the relative phase of the lower frequency modulation signal. Thus, measuring the phase of the modulated signal yields the same information as measuring the relative phase between the sidebands. Knowing the phase of a return OAM signal, with respect to an outgoing OAM signal, can be used to determine the distance traveled by the return signal.

Using a 1,500 MHz modulated HeNe laser as an example, the OAM wavelength would be approximately 200 mm. Another way to think of the 200 mm beat note is that every 200 mm, the phases of the two sidebands repeat. Since the path length is double, the phase of the return signal corresponds to a distance to the retroreflector of 100 mm. If the relative phase of one sideband is shifted by dispersion by 1 nm, or one part in 632.8 (0.0099 radians), the equivalent apparent distance to the retroreflector shift would be the same phase shift for the 100 mm wave, or 0.158 mm—which would be a huge error for high accuracy EDM instruments. This is illustrated graphically in Figures 1 and 2 of THE SPIE PAPER.

EDM instrument designers ignore the phase shifts introduced by the optics, because they measure the phase of a retroreflector at a first distance (typically a measured distance from the center of a gimbal mount to a fixed location) and make a software correction—assuming the correction is constant over the operating range of the instrument. It will be shown why that assumption may not be true.

EDM INSTRUMENT ARCHITECTURES

In order to understand the sources of EDM errors, it is useful to have a basic understanding of the operating principles of modern instruments. Most of the information on EDM instrument designs is found in the patent literature, which can be difficult to search. There are thousands of related US Patents, going back to the early 1960s, covering various aspects of EDM; including interferometers, total stations, laser trackers (which evolved from tracking laser interferometers to Absolute Distance Measurement [ADM] instruments), and now 3-D laser scanners (also called LIDAR). For example, a recent FARO patent [6] cites over 500 US patents and published applications as prior art. A small representative sample of the evolving architectures will be described.

Pioneering work on EDM was conducted by Froome and Bradsell at the National Physical Laboratory in Teddington, England. US Patents 3,521,956 (‘956) and 3,508,828 (‘828) (claiming priority to [or first disclosed] 1963 and 1965) [7, 8] disclosed inventions using cavity resonators to modulate the polarization angle, or amplitude, of light sources with measured distance accuracies of one part in 50,000 for ‘956 and 0.01 m at a distance of 10 km for ‘828.

Hölscher, at South African Inventions Development Corporation, disclosed in US Patent 3,365,717 (claiming priority to 1965) [9] modulating a gallium arsenide light emitting diode at a single frequency on the order of 75 MHz. The return signal is heterodyned down to 10 kHz by mixing with a reference oscillator. The phase is then measured for the 10 kHz signal, which can be used to determine a relative distance. The output beam was formed by a mirror, which has no dispersion, but the received beam is collected by a lens, which is dispersive.

Bagley et al., at Hewlett Packard, disclosed in US Patent 3,458,259 (filed in 1966) [10] methods for producing a two frequency laser using the Zeeman effect. The two frequencies produced a 500 kHz beat note signal. The beam was split into a fixed path and a measurement path. One detector counted the fringes for the fixed path and another detector counted the fringes for the measurement path. When the measurement path changed, the modulated signal was Doppler shifted, which produced a difference in the number of 500 kHz fringes counted between the two paths, corresponding to the change in distance. Note that the nominal 500 kHz fringe count made it possible to account for the direction of movement. This was the basis of the original HP 5525A interferometer introduced in 1971 [11].

Hewlett, at Hewlett Packard, disclosed in US Patent 3,619,058 (‘058) (filed in 1969) [12] methods for modulating a gallium arsenide electroluminescent diode at frequencies between 15 kHz and 25 MHz and measuring the phases of the return signals. By using a series of modulation frequencies, the absolute distance is determined.

Epstein, at Hewlett Packard, disclosed in US Patent 4,113,381 (filed in 1976) [13] combining Hewlett’s ‘058 invention with digital angle encoders and a theodolite to build the original Hewlett Packard Model 3810A “Total Station” surveying instrument, which was introduced in 1976 [11].

Payne and Parker (the author), at Associated Universities, disclosed in US Patent 5,455,670 (‘670) (filed in 1993) [14, 15] an architecture in which a laser is modulated at a first frequency of 1,500.000 MHz and the received signal is demodulated by a local oscillator (LO) at a second frequency, in which a Voltage-Controlled Crystal Oscillator (VCXO) is phase locked to the first frequency to run at 1,500.001 MHz. A fraction of the first and second frequencies are mixed to produce a nominal 1 kHz beat note, which is phase locked to a 1 kHz square wave reference frequency. The LO is mixed with the return signal to produce a 1 kHz intermediate frequency (IF) signal that retains the phase of the return beam. Another way to think of the demodulation signal is that it is a 1,500.000 MHz signal that is being linearly phase shifted by 2 π radians at 1 kHz.

The 1 kHz square wave reference frequency is generated by an Intel 8253 counter/timer chip. The 8253 also generates a 64 kHz A/D trigger impulse signal and a 1 kHz sync impulse. The 1 kHz IF signal is synchronously sampled 64 times per cycle by the A/D trigger, using direct memory access (DMA), starting at the sync impulse, and the phase is digitally determined relative to the 1 kHz sync impulse signal, i.e., there is no need to measure the phase of the reference clock since the timing of the A/D samples, the sync impulse, the 1 kHz square wave reference, and the LO are synchronized with the 8253.

This digital phase measurement technique is very similar to those later used by digital lock-in amplifiers, which produces a very accurate phase measurement. Note that the phase can be measured, under software control, 1,000 times per second, or averaged over a number of cycles—depending on the experiment being conducted. For example, a static measurement may be averaged over several hundred ms in order to reduce random noise due to atmospheric turbulence, and systematic 60 Hz line noise (having a 16.7 ms period), whereas measurements every ms may be used to measure vibrations by post processing to remove the high frequency and systematic noise.

Meier, at Leica, disclosed in US Patent 5,764,360 (claiming priority to 1995) [16] methods for modulating and demodulating, at a series of frequencies, the polarization angle of a laser beam in order to measure absolute distances. Instead of measuring the polarization angle, the modulation frequency is iteratively adjusted to null the difference in polarization angle between the outgoing and return beams, thus measuring the nulling frequencies to determine the absolute distance. It should be noted that circular polarization is simply simultaneous OAM in two orthogonal directions—and thus subject to the same dispersion.

Bridges et al., at Faro Laser Trackers, LLC (now FARO Technologies, Inc.), disclosed in US Patent 7,800,758 (claiming priority to 1999) [17] a fiber optic system in which the laser light is emitted from the end of a fiber optic placed precisely at the focal point of a convex lens, which produces a collimated outgoing beam. From the Helmholtz reciprocity principle, the retroreflected return beam returns into the end of the same optical fiber. Using a directional coupler, the return beam is coupled to a detector.

Bridges et al., at FARO Technologies, Inc., disclosed in US Patents 7,352,446 and 7,701,559 (claiming priority to 2004) [18, 19] methods for a digital phase measurement, similar to the Payne/Parker ‘670 patent, except that it measures a moving retroreflector. Once an initial absolute distance measurement is determined by using a series of modulation frequencies, the method tracks the phase wraps without needing to use multiple modulation frequencies again, i.e., it is an absolute distance meter (ADM) with the tracking speed of an interferometer.

Later 3-D laser scanners employ many of the same EDM techniques but, with scanners, speed is emphasized over accuracy, so dispersion in the optic is probably not a major concern. Nonetheless, readers may be interested in reviewing US Patent 5,988,862 to Kacyra et al., at Cyra Technologies (now assigned to Leica Geosystems AG) and US 8,064,046 to Ossig et. al., at FARO Technologies [20, 21]. The Ossig patent is particularly interesting in the fact that instead of sequentially changing the modulation frequencies to determine absolute distances, various frequencies are simultaneously used—as is done by some total stations.

It is believed that most commercially available laser interferometers, total stations, laser trackers, and laser scanners employ variations on these architectures. Pulsed time-of-flight systems can be modeled as having a continuous band of upper and lower sidebands, instead of a discrete pair of sidebands, that are subject to dispersion.

SOURCES OF ERROR DUE TO DISPERSION

The effects of dispersion due to the atmosphere are well known [22, 23], i.e., all EDM instruments correct for the group index of refraction n_g due to temperature, pressure, and humidity. However, it is noteworthy that, other than the author’s publications [1, 24], there is no mention in the literature of the errors in EDM due to dispersion of the optical elements internal to the instruments and retroreflectors. No known optics texts, design software, or optical component specifications (other than identifying the type glass) deal with temporally modulated light—which is possibly why it has been overlooked. Another reason may be that optical systems may be designed by optical engineers to gather the light on the detector, with little concern for the signal processing. Whereas the electronics engineer may not be concerned with the optics. It can be argued that a total systems approach is required in order to get the last nm of accuracy.

Section 3.1 and Figure 3 of THE SPIE PAPER explains how an OAM plane wave propagates through a converging lens in detail. The fundamental principles are illustrated in Figure 1. From Fermat’s principle [5], all light rays passing through the lens converge at the focal point in such a way that the transit times for all light rays from the source travel for the same length of time. In Figure 13, light ray 56A passes through air along the optical axis through the thickest, and slower, glass as ray 57A and through the shortest path 58A through air to the focal point.

Whereas light ray 56B passes through air along a parallel ray through the thinnest glass as ray 57B and through the longest path 58B to the focal point. The location of the focal point is determined by the front radius; index of refraction, n; and back radius.

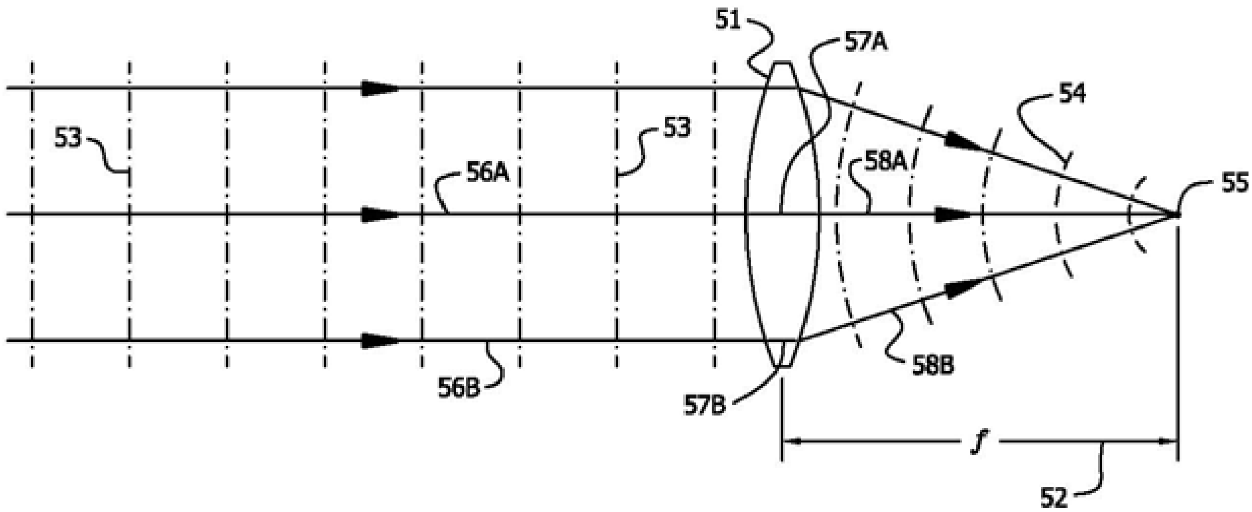

Figure 1

For a dispersive lens material, the speed of the modulated wave front through the glass is slower, due to the larger group index of refraction n_g. In other words, the phase of the modulated wave is retarded more for ray 57A than ray 57B, even though the carrier light rays all converge at the focus at the same time, i.e., in phase.

If the beam received by an instrument uniformly floods the lens and converges on a detector at the focal point, the phase of the electrical signal at the detector will be due to the integrated light on the detector, i.e., the phases of the different rays add as phasors. The averaging would be accounted for by the initial home measurement.

However, if the beam size is not uniform and floods the lens, the average phase can be dependent on which part of the lens the beam passes through. For example, the beam diverges with distance from the instrument. When the retroreflector is placed in the home position, the beam is small and passes mainly through the center of the lens. As the distance increases, the beam expands and fills more of the lens, which can shift the phases. This is analogous to fading in radio signals due to multipath induced fading—which not only changes the amplitude of the signal, but also the phase. For EDM instruments, this is an error in the measured distance.

One possible example of this multipath error may be shown in Figure 8 of a paper by Gross, presented at CMSC 2018 [25], which shows a plot of Distance Error vs ADM Distance, for a laser tracker before compensation. Of course there may be other explanations, but none was given in the paper.

Presumably, all laser trackers are compensated similarly. If in fact the error is due to dispersion; as long as the beam profile is repeatable, as a function of distance, the error would be completely corrected. However, if the beam profile were altered due to other causes, the compensation would not work correctly. For example, the beam profile could depend on such things as the retroreflector size, or the sharpness of the vertex point, e.g., a replicated retroreflector may have a dead zone at the vertex. A fingerprint, or other contamination on a retroreflector, or the objective lens, could produce regions where the light is scattered, and thus does not reach the detector.

Possible methods for testing an instrument would be to aline the instrument on a retroreflector and lock the pointing on the encoder angles, i.e., disable tracking. Then altering the beam profile by selectively blocking portions of the beam with stops, such as an iris, while recording the indicated distances. However, this would need to be done at the far range of the instrument in order to work with a flooded objective lens. Another possible problem is that in addition to modifying the beam geometry, the test also modifies the electrical signal power out of the detector, which as will be discussed below, can be an additional source of phase error due to the electronics.

An alternate test method, which can be conducted at close range and maintains constant power, is disclosed in US Patents 8,416,396 and 8,630,828 [24, 26] to Parker, at Parker Intellectual Property Enterprises, LLC. A retroreflector is translated orthogonal to the beam path at close range. Due to the lateral translation property of a retroreflector, the fixed outgoing beam will be returned as it is swept across the objective lens. Note that even if the translation is slightly tilted with respect to the orthogonal axis, it will be evident in the data.

Section 3.2 of THE SPIE PAPER discusses similar dispersion errors for spherical, also called cat's eye, retroreflectors. There are a number of spherical retroreflector designs in the US patent literature. A representative sampling includes US Patent 4,889,409 to Atcheson at Ball Corporation [27]; US Patent 5,126,879 to Ulbers at Hommelwerke GmbH [28]; and US Published Application 2009/0303592 to Oakley [29]. Ray tracings clearly show differences in path lengths through the glass, which cause the carrier wave front to retroreflect, but results in differing phases for the modulated wavefront. The orthogonal translation test of a spherical retoreflector vs a hollow retroreflector could be used to profile the phase shift due to the retroreflector design.

THE SPIE PAPER makes suggestions as to methods for designing lenses that avoid what the author calls OAM aberrations, to differentiate the problems from classical time invariant optical aberrations, at specific wavelengths. It also makes the argument that there is a need for optical design software that, in addition to modeling the classical time invariant design, also models the wave front for the OAM wave front. Such modeling would give rise to developing nomenclature and specifications for lenses that would be useful to EDM instrument designers.

OTHER SOURCES OF ERROR

In addition to the described OAM aberration, there are many other sources of error which should be mentioned. Rüeger and Burnside [22, 23] cover a number of them, but several will be pointed out in particular.

It is well known that a point source at the focal point of a lens will be collimated by the lens. Since most detectors, or fiber optic feeds to the detector, are at the focal point of a lens, particular attention must be given to light scattered by optical surfaces at the focal point. A small portion of the return beam that is scattered at the focal point is sent back to the retroreflector and returned again to the detector, i.e., makes multiple trips to the retroreflector. A plot of measured distance vs actual distance will exhibit a residual having a sine wave with a wavelength of the fundamental modulation wavelength. This is called the cyclic or short period error [22, 23].

A simple test to see if light is being scattered back is to aim the instrument at a test laser, of the same wavelength, which is located down range, in a darkened room. Hold the instrument pointing fixed and disable the instrument laser. Then aim the test laser along the same path as the instrument laser and into the instrument optics, i.e., simulate a return beam. Reflected light will be projected back to the test laser, producing an image of the optical surface on the face of the laser housing. A beam splitter and screen, or power meter, may be used to pick off the return light for closer examination.

Note that if the phase of the scattered beam happens to be in phase with the primary beam, there is no distance error. The worst case is when the phase of the scattered beam is orthogonal to the phase of the primary beam. For example, a noise signal of 1 part in 10^3 for a 1.5 GHz modulation signal would produce a maximum error of 0.0159 mm.

Feedback into the laser is also well known to be a potential problem. There are methods for measuring the phase shift due to feedback into the laser, but they are beyond the scope of this paper.

Leakage of microwave signals within the electronics may give rise to mixing which can produce small noise signals, of a constant phase, at the IF. This is explained in detail by Rüeger in section 12.2.1 [22]. One test for leakage is to aim the instrument at the sky in order to receive zero return signal. The instrument should measure zero signal amplitude and the phase should be random. If the phase is steady, there is a good chance it is due to a leakage signal.

For instruments that use the Payne/Parker ‘670 [14, 15] digital phase measurement technique, a constant leakage component can be removed from the measured signal by subtracting the constant leakage phasor from the measured phasor. The digital phase measurement technique can also detect higher harmonics of the IF. For example, the return IF signal would ideally be a pure sinusoidal wave at the IF frequency. However, non-linearities, such as clipping, would give rise to higher harmonic components which should be investigated—or built into the operating software to produce a warning.

Polarization modulation techniques can be particularly sensitive to polarization sensitivity of the retroreflector. A simple test is to rotate the retroreflector about the optical axis while recording the indicated distance. It may be prudent to orient the retroreflector at a constant angle about the optical axis.

For high precision measurement of phase, particular care must be taken for the electronics. In particular, the electronics may be amplitude sensitive. For example, coupling capacitors can be non-linear, which could produce a phase shift. Sub milliradian phase shifts are not a concern for most electronic circuits, but are critical for EDM instruments. Testing can be conducted using digital lock-in amplifiers, but this is beyond the scope of this paper.

Inserting neutral density filters in the beam can introduce problems, e.g. the thickness, and tilt, of the filter introduces an additional phase shift. A simple test is to record the amplitude and distance to a fixed retroreflector. Fog the mirrors of the retroreflector by placing a cup of hot water under the retroreflector (or exhale on the retroreflector). The fog on the mirror will scatter the beam and the amplitude should drop to zero. Then remove the cup and record the amplitude and distance as the fog evaporates. The signal level will rise over several seconds, which will produce a nice curve of distance vs power. If there is a residual signal while the mirror is fogged, this would be a strong indication of leakage.

One way to variably attenuate the return beam is to use an attenuated hollow retroreflector [30], in which one (or more) of the first surface mirrors are replaced by a glass neutral density filter. Since all beams undergo three reflections, one reflection will be from the front surface of the filter (with the remaining energy being absorbed), which greatly reduces the efficiency of the retroreflector. Reflections from the glass are polarization sensitive, so variable attenuation may be obtained by rotating about the optical axis.

There are presently no industry standards for specifications for the servo systems and tracking accuracy. A simple suggestion is disclosed in US Patent 7,856,334 [31] to Parker, at Parker Intellectual Property Enterprises, LLC which uses various harmonic motion oscillators to produce motions of a retroreflector, e.g., a pendulum.

CONCLUSIONS

Even though commercially available EDM instruments are extremely accurate, there are some applications in which the best you can get is not overkill. This paper brings some of the subtler sources of error to the reader's attention.

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